Chromatic Transformation Labels for the Symmetric Triads

Additions to David Kopp’s Chromatic Transformation System to Accommodate the Dissonant Triads

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Abstract—David Kopp’s Chromatic Transformation System is a thorough analytical system of labels for transformations between major and minor triads. Kopp extends Brian Hyer’s D, P, L, R, and I scheme via refinements from Richard Cohn’s concept of maximally smooth cycles within the PLR transformations. While Kopp’s neo-Riemannian theory is immensely useful for analyzing chromaticism in tonal music, the system does not accommodate transformations involving the intervallically symmetric triads, viz., augmented or diminished triads. Both diminished and augmented triads can assume functional roles in harmony, even though they cannot be tonal destinations, hence there is value for their potential inclusion in an expanded transformation network. This paper discusses the need for an extension of Kopp’s Chromatic Transformation System to encompass analyses involving this neglected triad group, systematically develops and presents five additional transformational symbols consistent with the philosophies of Kopp’s system, and applies them in analyses of music by Schubert, Chopin, Liszt, and Wagner.

Index Terms—Harmony, Chromaticism (Music), Music Theory

I. INTRODUCTION

It is without doubt that analytical attempts to explain the chromatic music of the nineteenth-century have sparked debate, exploded in a plethora of contrasting views, and have often fallen short of providing satisfactory explanations which yield any useful or profound insights about the music in question. Traditional theories regarding diatonic progressions, and many functionally chromatic ones such as resolutions of secondary dominants, Neapolitan chords, and augmented-sixth chords, have been well-settled and accepted, but chromatic mediants have long been theoretical mysteries. Few theorists have succinctly presented such a useful analytical tool that encompasses a musical space including chromatic mediants as David Kopp in his “Chromatic Transformations in Nineteenth-Century Music.” Kopp’s ambitious work lends itself to analysis of many musical examples which previously were viewed as demonstrative of the breakdown of tonality, the limitations of traditional harmonic theory, or in general, explicable only in terms of voice leading and melodic content, rhythm, texture, and extra-musical elements, but apparently lacking coherence when the harmonies were taken out of the musical or extra-musical context.

Kopp’s general philosophy is that common-tone relationships are the connective tissues which allow passages of chromatic music, especially those characteristic of nineteenth-century compositions, to exhibit rational coherence in terms of the harmonic transformations themselves and not rely on the “crutches” of other contextual explanations in chromatic music. He presents a set of transformational labels which open a new frontier of the aforementioned analysis possibilities. However, Kopp fails to address some important sonorities which become of increasing importance in later nineteenth-century music. The Chromatic Transformation System (CMT) is restricted to a musical space consisting almost exclusively of the consonant triads, viz., those that are major and minor. We are left with some “nagging questions that need answers.” [2] Kopp’s CMT cannot account for augmented or diminished triads, which are important, and increasingly independent sonorities in music of the same time period of which Kopp addresses. Kopp also fails to satisfactorily justify transformations involving seventh chords or other extended tertian harmonies. Chord sevenths are treated as mere appendages to triads and do not play significant roles in transformations. This paper will focus exclusively on augmented and diminished triads and attempt to extend Kopp’s system to account for them.

II. SEGREGATION OF TRIADIC SPACES

Because David Kopp’s CMT operates on the limited set of the consonant triads, major and minor, it is tempting to graft the dissonant triads directly into the same musical “space.” Unfortunately, the dissonant triads fail in many ways to exhibit functional behavior that can be described in similar terms to their asymmetric cousins. For example, in his early theoretical views, Hugo Riemann claimed that the only directly intelligible root relationship in tonal progressions is the descending fifth. But what does it mean for an augmented or diminished triad to have root motion by a fifth? Such questions demonstrate the inappropriate application of well-defined tonal terminology to the dissonant triads. Similarly, root motion by major third for augmented triads presents a particular problem of having no modification of pitch class. Thus, for both practi-
cal\textsuperscript{3} and theoretical reasons, I will propose that the consonant and dissonant triads be segregated into distinct mathematical sets, each with its own terminology, internal transformational functions, but with common, interrelated mapping functions to translate between the two.

A. Definitions of Triad Spaces

To facilitate further discussion, the triad sets should be clearly defined for true mathematical rigor to be applicable. First, the general sets of consonant triads will consist of major and minor triads, while the consonant set will consist of augmented and diminished triads. Then the subsets for each quality of triad will be defined. For the following definitions, \( \tau \) will represent a triad of any quality.

1) Consonant Triads (major and minor):

Definition Let \( \mathbb{C} \) be the set consisting of all triads which are major or minor:
\[
\mathbb{C} = \{ \tau \, | \, \tau \text{ is any consonant triad (major or minor)} \}.
\]

Definition Let \( \mathbb{m} \) be a subset of \( \mathbb{C} \) where every element \( \tau \in \mathbb{m} \) is a minor triad:
\[
\mathbb{m} = \{ \tau \, | \, \tau \text{ is any minor triad} \}.
\]

Definition Let \( \mathbb{M} \) be a subset of \( \mathbb{C} \) where every element \( \tau \in \mathbb{M} \) is a major triad:
\[
\mathbb{M} = \{ \tau \, | \, \tau \text{ is any major triad} \}.
\]

2) Dissonant Triads (augmented and diminished):

Definition Let \( \mathbb{D} \) be the set consisting of all triads which are augmented or diminished:
\[
\mathbb{D} = \{ \tau \, | \, \tau \text{ is any dissonant triad (aug. or dim.)} \}.
\]

Definition Let \( \mathbb{a} \) be a subset of \( \mathbb{D} \) where every element \( \tau \in \mathbb{a} \) is an augmented triad:
\[
\mathbb{a} = \{ \tau \, | \, \tau \text{ is any augmented triad} \}.
\]

Definition Let \( \mathbb{d} \) be a subset of \( \mathbb{D} \) where every element \( \tau \in \mathbb{d} \) is a diminished triad:
\[
\mathbb{d} = \{ \tau \, | \, \tau \text{ is any diminished triad} \}.
\]

B. Transformational Separation of Spaces

As mentioned, the goal of the above definitions is to facilitate both mathematical discussion as well as clearly define the separations between Kopp’s system and the ideas presented in this paper. The following presentation of analytical tools for labeling transformations involving dissonant triads will not affect the existing labels proposed by Kopp. The CMT developed by Kopp will continue to operate exclusively within the \( \mathbb{C} \) set, but the ideas presented here will deal not only with transformations involving the \( \mathbb{D} \) set, but also transformations between the two: \( \mathbb{C} \leftrightarrow \mathbb{D} \).

As shown in Figure 1, the consonant triads are transformed with Kopp’s system of labels and the dissonant triads are separated into their own space. Some mapping functions are required to translate between the two spaces.

![Fig. 1. Distinction between Consonant and Dissonant Triad Spaces](Image 312x595 to 563x651)

III. AUGMENTED TRIADS

Before developing transformational labels which account for the augmented triads, let us begin our discussion by exploring their basic properties. The following two properties are intrinsic properties of augmented triads, i.e., these are not empirically observed uses.

A. Interval Symmetry

Augmented triads are symmetrical, viz., they contain symmetric interval content about all rotations. That is, the only interval present in an augmented triad is a major third. This leads to the interesting consequence that any rotation (inversion) of an augmented triad yields an equivalent pitch-class:
\[
C^+ \equiv E^+ \equiv G^{#+}
\]
where the equivalence relation denotes the equivalence of pitch content. A consequence of this property is that there are only four enharmonically different augmented triads that exist: \( C^+ \), \( D^#^+ \), \( D^+ \), and \( E^{#+} \).

B. Closeness to Consonant Triads

Augmented triads are a maximally smooth transformation away from triads in the consonant set. That is, a semi-tone motion of \textit{any} pitch in an augmented triad yields a consonant triad. For example, Table I shows how raising or lowering any pitch in a \( C^+ \) triad results in a consonant triad.

<table>
<thead>
<tr>
<th>Chord Member Movement</th>
<th>Resultant Triad</th>
</tr>
</thead>
<tbody>
<tr>
<td>#5</td>
<td>C</td>
</tr>
<tr>
<td>#5</td>
<td>F</td>
</tr>
<tr>
<td>#3</td>
<td>A</td>
</tr>
<tr>
<td>#3</td>
<td>f#</td>
</tr>
<tr>
<td>#1</td>
<td>E</td>
</tr>
<tr>
<td>#1</td>
<td>e#</td>
</tr>
</tbody>
</table>

\textsuperscript{3}It is quite impractical to use, for day-to-day analysis, a mathematically complete set of relations that encompass both consonant and dissonant triad sets. For such a system, see David Lewin’s “Generalized Musical Intervals and Transformations.” Kopp’s ideas greatly hone in transformational theory to a few specific but clearly recognized transformations which are useful for practical analysis.
IV. TRANSFORMATIONS INVOLVING AUGMENTED TRIADS

In the development of transformational functions involving augmented triads, two underlying philosophies must be considered. First, in keeping with Kopp’s proclamation of the almighty common-tone relation, fundamentally recognized transformations must involve common-tones. Second, for the practicality of analysis, any additional labels which will augment⁴ the CMT should be as simple as possible without sacrificing concepts of function on the altar of Occam’s Razor.

A. Mapping Functions

As a first priority, mapping functions between the consonant and dissonant triads are required. To begin, let us consider the possible ways an augmented triad can be transformed into a consonant triad. Table I lists six possible transformations that will produce a consonant triad from an augmented one through only a semi-tone motion of any member of the augmented chord. However, given a major triad, how can we generate an augmented triad? Clearly, the minor third interval must be expanded to a major third by modifying it with a 5. Additionally, given a minor triad, the minor third interval must be expanded to a major third by raising the root. Any of the other possibilities in Table I would require some root motion within the C space to produce the augmented triad.

Raising or lowering the fifth of major and augmented triads respectively is essentially an invertible relationship; one yields the other. Similarly, lowering or raising the root of a minor or augmented triad creates a cycle of length two that is reminiscent of the leittonwechsel concept. These are the simplest two relationships, I believe, between consonant and augmented sets since other relationships require what appear to be compound relations in root motion. This leads to the definition of mapping functions for major and minor triads to augmented triads.

First, it will be noted that, in an effort to visually and mentally distinguish between the consonant transformation labels as Kopp presents them, Greek letters will be used to represent interaction with dissonant triads⁵.

The following function⁶ τ, with respect to transformations between augmented triads and major triads, is defined as:

$$ \tau(\tau^\circ) = \tau. $$

That is, when τ is a major triad (element of M), the τ transformation raises the fifth to produce an augmented triad with the same root. When τ is an augmented triad (element of a), τ lowers the fifth to produce a major triad of the same root. According to this definition, τ is a sign-reversing involution⁷, that is,

$$ \tau(\tau(\tau(\tau^\circ))) = \tau. $$

Similarly, the function α with respect to transformations between augmented triads and minor triads is defined as:

$$ \alpha(\tau) = \begin{cases} \tau_1 & : \tau \in M \\ \tau_2 & : \tau \in a \end{cases} $$

That is, when τ is a minor triad (element of m), the α transformation lowers the root to produce an augmented triad. When τ is an augmented triad (element of a), α raises the root to produce a minor triad. According to this definition, α is also a sign-reversing involution, that is,

$$ \alpha(\alpha(\tau)) = \tau. $$

One might wonder why a single transformational label is not proposed that works for both major and minor chords since there is only one way to transform a major or minor triad into an augmented one. However, it becomes problematic when the process is reversed. An augmented triad that is to be translated back to the consonant domain has the choice of becoming a major or minor triad. Thus it is necessary to retain separate functions that represent each. Note that the choice of upper- and lower-case alpha parallels the convention used by Kopp for major and minor distinction.

To illustrate an “augment” transformation in real music, observe the opening measures of Schubert’s Die Sterne (D. 684) in Figure 2. The E♭ triad is transformed to an E♭ by raising the B♭ to a B♯. A Neo-Riemannian labeling using the alpha transformation would be:

$$ E♭ \xrightarrow{A} E♭ $$

![Fig. 2. Schubert - Die Sterne (D. 684) mm. 1-2](image)

B. Transformations for Augmented Triads

Kopp’s system of transformations is heavily centered in relations which are considered to be “fundamental” relations. Just as Riemann was so convinced of the utmost importance of the fifth relation, so Kopp has introduced to us the importance of the third relation with the relative and mediatant functions. The philosophical question is, what might be considered fundamentally heard transformations in augmented triads? Certainly their place in functionally tonal harmonic progressions has always been subservient to the consonant triads, and they have historically been treated as mere embellishments of tonally functional chords or simply as agents of harmonic color. Additionally, the concept of “root motion” for augmented triads is questionable itself because there are no common tones between enharmonically distinct augmented triads and root.

⁴Pun most wholeheartedly intended.
⁵These should not be confused with Bryan Hyer’s “4D transformational scheme” which uses δ to represent dominant movement, λ for leittonwechsel and ρ for relative functions.
⁶It is regrettable that the capital Greek letter “alpha” is our letter A. Nevertheless, it makes the most sense to “Augment” a triad.
⁷A sign-reversing involution is a mathematical function which is its own inverse and hence exhibits reciprocal accretion.
motion by a third or augmented fifth produces chords which are enharmonically equivalent.

For these reasons, only one transformation will be proposed for triads located in the $A$ set. The Rotate function $\rho$ will represent root motion down by third between augmented triads.

$$\rho(\tau) : A \rightarrow A = \tau \begin{bmatrix} 5 & 1 \\ 3 & 5 \\ 1 & 3 \end{bmatrix}$$

The inverse rotate function $\rho^{-1}$ represents root motion up by major third:

$$\rho^{-1}(\tau) : A \rightarrow A = \tau \begin{bmatrix} 5 & 3 \\ 3 & 1 \\ 1 & 5 \end{bmatrix}$$

1) Compound Relationships: This rotational function supplies us with a concept of “movement” within the dissonant augmented set which preserves the philosophy of common-tone tonality. This “root motion” allows an augmented triad to be moved and then “un-augmented” to represent motion within the consonant set. Consider again the example in Table I. Using the $\rho$ function, we can now represent all possible maximally smooth transformations from an augmented triad to a consonant one via a compound relationship as shown in Table II.

### TABLE II
**Modifying Chord Members in a $C^+$ Triad**

<table>
<thead>
<tr>
<th>Chord Member Movement</th>
<th>Transformation</th>
<th>Resultant Triad</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\flat 5$</td>
<td>$A$</td>
<td>$C$</td>
</tr>
<tr>
<td>$\flat 5$</td>
<td>$\rho^{-1}\alpha$</td>
<td>$A\flat m$</td>
</tr>
<tr>
<td>$\flat 3$</td>
<td>$\rho^{-1}A$</td>
<td>$A\flat m$</td>
</tr>
<tr>
<td>$\flat 3$</td>
<td>$\rho\alpha$</td>
<td>$E\flat m$</td>
</tr>
<tr>
<td>$\flat 1$</td>
<td>$\rho A$</td>
<td>$E\flat m$</td>
</tr>
<tr>
<td>$\flat 1$</td>
<td>$\alpha$</td>
<td>$C\flat m$</td>
</tr>
</tbody>
</table>

Note that each of these compound relationships have identities related to Kopp labels. The identities in Table III demonstrate the transformation $\tau_1 \xrightarrow{\text{Compound Relationship}} \tau_2$ with an equivalent Kopp transformation: $\tau_1 \xrightarrow{\text{Kopp}} \tau_2$.

However, the relations in Table III are not fully interchangeable since the $A$ and $\alpha$ functions can only be applied to major or minor triads respectively while the Kopp labels can be used for either.

2) New Interpretative Possibilities: An example of how these compound relationships can be used to provide insight into a progression is illustrated in another excerpt from Schubert’s *Die Gebüschke* (D. 646) shown in Figure 3. Here we see the progression: $G \rightarrow G^+ \rightarrow E\flat m$. Three possible interpretations where the augmented triad is spelled differently exist:

$$G \xrightarrow{A} G^+ \xrightarrow{A\flat r} E\flat m$$

$$G \xrightarrow{A\flat r} D^+_4 \xrightarrow{\alpha} E\flat m$$

### TABLE III
**Equivalent Identities for Augmented Transformations**

<table>
<thead>
<tr>
<th>$\tau_1$</th>
<th>Transformation</th>
<th>$\tau_2$</th>
<th>Equivalent Relation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C$</td>
<td>$A\flat A$</td>
<td>$Ab$</td>
<td>$M$</td>
</tr>
<tr>
<td>$C\flat m$</td>
<td>$\alpha\rho A$</td>
<td>$Ab\flat m$</td>
<td>$M$</td>
</tr>
<tr>
<td>$C$</td>
<td>$A\rho^{-1}A$</td>
<td>$E$</td>
<td>$M^{-1}$</td>
</tr>
<tr>
<td>$C\flat m$</td>
<td>$\alpha\rho^{-1}\alpha$</td>
<td>$e\flat m$</td>
<td>$M^{-1}$</td>
</tr>
<tr>
<td>$C\flat m$</td>
<td>$\alpha\rho^{-1}A$</td>
<td>$E\flat m$</td>
<td>$r$</td>
</tr>
<tr>
<td>$C$</td>
<td>$A\rho\alpha$</td>
<td>$a\flat m$</td>
<td>$r$</td>
</tr>
<tr>
<td>$C$</td>
<td>$A\rho^{-1}A$</td>
<td>$f\flat m$</td>
<td>$F$</td>
</tr>
<tr>
<td>$C\flat m$</td>
<td>$\alpha\rho A$</td>
<td>$G$</td>
<td>$F^{-1}$</td>
</tr>
<tr>
<td>$C$</td>
<td>$A^2$</td>
<td>$C$</td>
<td>$I$</td>
</tr>
<tr>
<td>$C\flat m$</td>
<td>$\alpha^2$</td>
<td>$C\flat m$</td>
<td>$I$</td>
</tr>
<tr>
<td>$C^+$</td>
<td>$\rho^3$</td>
<td>$C^+$</td>
<td>$I$</td>
</tr>
<tr>
<td>$C^+$</td>
<td>$\rho^{-3}$</td>
<td>$C^+$</td>
<td>$I$</td>
</tr>
<tr>
<td>$C$</td>
<td>$A\alpha$</td>
<td>$C\flat m$</td>
<td>$S$</td>
</tr>
<tr>
<td>$C\flat m$</td>
<td>$\alpha A$</td>
<td>$B$</td>
<td>$S$</td>
</tr>
</tbody>
</table>

Although the first choice correctly handles the spelling of the $G^{+}$ triad, the prominence of the $D^+_4$ and its leading-tone motion to the $E$ suggests that the second choice may be a better interpretation while the last possibility demonstrates a dominant function to the $e\flat m$ triad through a fifth change. Of course, the overall progression is an $r$ relation, so the first choice may be more representative as a hierarchical demonstration of that relative motion. Since the choices exist, each could be used by the analyst to highlight different aspects of a progression. A traditional harmonic analysis may claim $I \rightarrow V^+/vi \rightarrow vi$. But these new transformational labels may shed more light on the details of the progression and allow more interpretive possibilities.

### V. Transformations Involving Diminished Triads

Diminished triads offer additional challenges since history has treated them with more favor than the mysterious augmented triads. This is partially due to the fact that in the major mode, $ii^5$ is diatonic as is also $ii^6$ in minor—they arise naturally in diatonic progressions while augmented triads are not diatonic in any key.

There are two important items to understand regarding diminished triads. First, they often serve a dominant function by tonicizing the following chord through a resolved leading-tone root motion. Second, diminished triads do not exhibit fully symmetrical behavior as augmented triads do. However, fully diminished seventh chords contain the symmetric intervals for this property to exist.

From the first point, one might argue that a transformational label which indicates a leading-tone resolution/dominant function for a diminished triad should exist to account for what we may recognize as a fundamental transformation. However, passages that include such traditional progression

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8We can add to the author of Ecclesiastes who said “In this meaningless life of mine I have seen both of these: the righteous perishing in their righteousness, and the wicked living long in their wickedness.” So too the augmented triad is dealt a similar injustice.
can be easily accounted for by invoking an implied root below the diminished triad and then using a dominant or fifth-change transformation. For example, the progression \( B^\circ \rightarrow C \) can be written as \( B^\circ (G^7) \xrightarrow{D} C \) which accurately portrays the dominant function of the \( B^\circ \) in terms of both transformational theory and traditional harmonic ideas. Thus, I will propose no new symbols to represent the dominant function of diminished triads.

### A. Mapping Functions

As with the augmented triads, let us examine how a diminished triad might be generated from a major and minor one. From a major triad, the major third interval must be diminished into a minor third via \( \#1 \). From a minor triad, the appropriate action is \( \flat5 \). Conversely, from a diminished triad, these are also the only two ways to transform to a consonant triad by only semi-tone motion of a chord member. Thus, in the spirit of the previously-developed transformational labels it is natural to define the function \( \Delta \) with respect to transformations between diminished triads and major triads as:

\[
\Delta(\tau) = \begin{cases} 
\tau^\sharp & : \tau \in \mathcal{M} \\
\tau^\flat & : \tau \in \mathcal{D}
\end{cases}
\]

That is, when \( \tau \) is a major triad (element of \( \mathcal{M} \)), the \( \Delta \) transformation raises the root to produce a diminished triad. When \( \tau \) is a diminished triad (element of \( \mathcal{D} \)), \( \Delta \) lowers the root to produce a major triad. According to this definition, \( \Delta \) exhibits reciprocal accretion, that is,

\[
\Delta(\Delta(\tau)) = \tau.
\]

The function \( \delta \) with respect to transformations between diminished triads and minor triads is defined as:

\[
\delta(\tau) = \begin{cases} 
\tau^\flat5 & : \tau \in \mathcal{M} \\
\tau^\sharp5 & : \tau \in \mathcal{D}
\end{cases}
\]

That is, when \( \tau \) is a minor triad (element of \( \mathcal{M} \)), the \( \delta \) transformation lowers the fifth to produce a diminished triad. When \( \tau \) is a diminished triad (element of \( \mathcal{D} \)), \( \delta \) raises the fifth to produce a minor triad. According to this definition, \( \delta \) exhibits reciprocal accretion, that is,

\[
\delta(\delta(\tau)) = \tau.
\]

For an example of such transformations in real music, we will again visit Schubert in the concluding measures of *Die Gebüsche* (D. 646) shown in Figure 4. Here we see the progression: \( G \rightarrow g^\circ \rightarrow G \) which, invoking the parallel mode, can be represented as

\[
G \xrightarrow{P^\flat} g^\circ \xrightarrow{D^\flat} G.
\]

This makes some intuitive sense since the \( g^\circ \) chord is not acting in a dominant fashion, but is rather an embellishment of tonic.

### B. Transformations for Diminished Triads

It is no secret that diminished triads are closely related to fully-diminished seventh chords. In fact, we often see progressions such as \( d^\circ \rightarrow g^\circ7 \rightarrow a\tilde{m} \), where the \( d^\circ \) is “repelled” as part of a \( g^\circ7 \), which acts as a leading-tone to \( a\tilde{m} \). To account for such cases, it is tempting to extend the currently discussed musical spaces to include fully-diminished seventh chords. However, due to the enormous increase in complexity required to wield transformational theory against seventh chords, we will only discuss transformations which involve diminished triads.

For a progression as mentioned above, we will extend the functionality of the rotate function \( \rho \) to represent a root motion down by minor third when applied to diminished triads and retains the quality. This means that through a \( \rho \) transformation, a fully-diminished seventh chord will inevitably be covered, thus partially accounting for them. Similarly, \( \rho^{-1} \) will be root motion up by minor third, again tracing out a fully-diminished seventh chord.
\[
\rho(\tau) = \begin{cases} 
\tau \to M3 \downarrow & : \tau \in \mathbb{A} \\
\tau \to \bar{m}3 \downarrow & : \tau \in \mathbb{D}
\end{cases}
\]

\[
\rho^{-1}(\tau) = \begin{cases} 
\tau \to M3 \uparrow & : \tau \in \mathbb{A} \\
\tau \to \bar{m}3 \uparrow & : \tau \in \mathbb{D}
\end{cases}
\]

1) Compound Relationships: As with the transformations involving augmented triads, the functions transforming with diminished triads can be combined in multiple ways equivalent to the standard Koppian transformations. Table IV shows a list of all combinations of the \(\Delta, \delta\) and \(\rho\) functions.

<table>
<thead>
<tr>
<th>(\tau_1)</th>
<th>Transformation</th>
<th>(\tau_2)</th>
<th>Equivalent Relation</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>(\Delta \rho \Delta)</td>
<td>A</td>
<td>m</td>
</tr>
<tr>
<td>C/m</td>
<td>(\delta \rho \bar{m})</td>
<td>a</td>
<td>m</td>
</tr>
<tr>
<td>C</td>
<td>(\Delta \rho^{-1} \Delta)</td>
<td>E</td>
<td>m</td>
</tr>
<tr>
<td>C/m</td>
<td>(\delta \rho^{-1} \delta)</td>
<td>e</td>
<td>m</td>
</tr>
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<td>C</td>
<td>(\Delta \rho^{-1} \Delta)</td>
<td>c</td>
<td>m</td>
</tr>
<tr>
<td>C/m</td>
<td>(\delta \rho\delta)</td>
<td>A</td>
<td>m</td>
</tr>
<tr>
<td>C</td>
<td>(\Delta \rho \delta)</td>
<td>B</td>
<td>m</td>
</tr>
<tr>
<td>C/m</td>
<td>(\delta \rho^{-1} \Delta)</td>
<td>D</td>
<td>D</td>
</tr>
<tr>
<td>C</td>
<td>(\Delta^2)</td>
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<td>C/m</td>
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<td>(\rho^4)</td>
<td>C^m</td>
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<td>(\rho^{-4})</td>
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<td>m</td>
</tr>
<tr>
<td>C</td>
<td>(\Delta\delta)</td>
<td>C</td>
<td>m</td>
</tr>
<tr>
<td>C/m</td>
<td>(\delta \Delta)</td>
<td>B</td>
<td>S</td>
</tr>
</tbody>
</table>

2) A Note about Fully-Diminished Seventh Chords: As mentioned earlier, it is tempting to graft fully-diminished seventh chords into this new labeling system which accounts for diminished triads but the technical difficulties are readily apparent. However, the \(\rho\) function does in some way indirectly account for them since every application of \(\rho\) to a diminished triad will introduce the fourth chord member of a fully-diminished seventh chord. The farthest I will go here is to say that we may find it useful to allow \(\rho\) to rotate fully-diminished seventh chords. Then, ignoring the chord seventh in the spirit of Kopp, we can apply the diminished transformations to the root triad.

VI. SUMMARY OF THE EXPANDED CHROMATIC TRANSFORMATION SYSTEM

It is important to realize that the augment and diminish functions are in some ways inverses of each other. Combinations of augmenting functions can produce \(M, M^{-1}\), and \(R\) while diminishing functions can create \(m, m^{-1}\), and \(r\). This is, of course, a result of the major third and minor third symmetry found in the augmented and diminished triads respectively. Finally, both can produce the \(F, F^{-1}\) and \(S\) transformations. All these relations in Tables III and IV show that every label that Kopp’s CMT contains can be represented as a compound relationship involving augmented or diminished triads. This demonstrates the compliance of the new labels with Kopp’s philosophical approach. Additionally it makes sense that these relationships are compound since it is well established that transformations between the consonant triads are those which are fundamentally recognized in tonal music; any other route that traces out such a transformation but invokes other chords, especially dissonant ones, should be considered subsidiary.

The preceding sections have introduced five additional labels to the CMT. David Kopp presents a table of properties of his labels in “Chromatic Transformations in Nineteenth-Century Music” that is reproduced in an expanded edition that includes these new labels in Table V.

VII. THREE ANALYTICAL EXAMPLES

Not in keeping with the traditions set forth by Heinrich Schenker and David Kopp who presented five analyses utilizing their theoretical developments, this paper will only examine three short excerpts from Schubert, Liszt and Wagner. These will be reminiscent of Kopp’s graphical network analyses, but with the added benefit of accommodating the dissonant triads.

A. Schubert, Die Gebüische (D. 646)

As two examples of augmented and diminished functions having already been drawn from this Schubert lied, a more thorough analysis seems appropriate. The first 20 bars are printed in Figure 11. The first three measures gorgeous piece open directly with motion from tonic to an augmented triad which resolves to the leittonwechsel. As mentioned previously, the three possibilities of analysis are for \(G \to G^+ \to e|m\) are \(A - Ar, A|p - \alpha,\) and \(A|p - \alpha - AF\). Due to the strong presence of the \(D_7^2\) in the base and its leading-tone resolution to \(e|m\), I will propose the second possibility, which invokes the leading-tone concept through the \(\alpha\) function which raises the root of an augmented triad to produce a minor triad. The introduction ends with a figure sweeping from a \(G\) through a \(g^0\) and back to a \(G\) at which point the voice comes in on the dominant. This piano introduction is graphed in Figure 5.

![Fig. 5. Schubert, Die Gebüische (D. 646) Introduction (mm. 1-6)](image-url)

Once the voice comes in at measure 6, the \(G\) tonic is firmly established by another \(D_7\) in measure 8 and back to \(G\) in measure 9. The remainder of the piece is quite traditional in terms of harmonic language, except for a repeat of the augment function resolving to a relative in measures 15-17 where Schubert uses a \(C \xrightarrow{\alpha} C^+ \xrightarrow{\alpha} e|m\), which is exactly the same progression found in the opening measures transposed down a fifth.
B. Liszt, Harmonies poétiques et religieuses (1833)

As an example of Liszt’s use of common tones to create augmented triads which can be used to pivot into other harmonic regions, let us examine a few measures of Harmonies poétiques et religieuses as shown in Figure 12. This section is reduced in Figure 7. Here we see a $D♭$ chord which is augmented to produce a $D♭+1$ which in turn transforms into a $D$. Of course the overarching movement is a $SP$, but we succinctly use the augment function to show the details. See Figure 6.

C. Wagner, Tannhäuser, act I, sc. 1, “Venusberg” (1845)

Wagner also uses the augmented triad to create harmonic color while providing forward momentum which ultimately prolongs a harmony. We will examine his Tannhäuser, act I, scene 1, “Venusberg,” measures 81-88 shown in Figure 13. A reduction is provided in Figure 8.

VIII. Conclusion

The Chromatic Transformation System proposed by David Kopp has been criticized for its lack of accommodation for the dissonant triads. To rectify this situation, this paper has proposed five new transformational labels which, when applied to analysis, can not only reveal to us interesting aspects of harmonic progression, but also give us a sense of satisfaction in knowing that our analyses can take into account every triadic progression. For example, the following excerpt (Figure 10) from the Chopin’s second cello sonata is analyzed by Kopp. But left in a state of lonely despair is the $d♭7$ chord which has no transformational label attached to it. Now with the tools at hand, we can analyze this progression as:

$$A♭ \xrightarrow{DΔ} d♭7 \xrightarrow{δE} G7.$$
REFERENCES


Fig. 11. Schubert - Die Gebüße (D. 646) mm. 1-20
Fig. 12. Liszt - Harmonies poétiques et religieuses (1835) mm. 46-49

Fig. 13. Wagner, Tannhäuser, act I, sc. 1, “Venusberg” (1845) mm. 81-88
### TABLE V

**SOME ASPECTS OF THE CHROMATIC TRANSFORMATION SYSTEM (EXPANDED)**

<table>
<thead>
<tr>
<th>Mode relation</th>
<th>Dominant ($D$)</th>
<th>Fifth-change ($F$)</th>
<th>Mediant ($M/m$)</th>
<th>Relative ($R/r$)</th>
<th>Identity ($I$)</th>
<th>Parallel ($P$)</th>
<th>Augment ($A/\alpha$)</th>
<th>Diminish ($\Delta/\delta$)</th>
<th>Rotate ($\rho^{(+/\circ)}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{Root relation}$</td>
<td>plain</td>
<td>change</td>
<td>plain</td>
<td>change</td>
<td>plain</td>
<td>change</td>
<td>change</td>
<td>plain/semi-tone</td>
<td>semi-tone/plain</td>
</tr>
<tr>
<td>$\text{Common tones}$</td>
<td>perfect fifth</td>
<td>perfect fifth</td>
<td>major/minor third</td>
<td>major/minor third</td>
<td>prime</td>
<td>prime</td>
<td>two</td>
<td>two</td>
<td>three/two</td>
</tr>
<tr>
<td>Varieties</td>
<td>one</td>
<td>one</td>
<td>two</td>
<td>two</td>
<td>four</td>
<td>two</td>
<td>one</td>
<td>one</td>
<td>one</td>
</tr>
<tr>
<td>$\text{Harm. range and outcome}$</td>
<td>$1\flat \leftrightarrow 1#$</td>
<td>$4-2\flat \leftrightarrow 2-4#$</td>
<td>$4-3\flat \leftrightarrow 3-4#$</td>
<td>$1-0\flat \leftrightarrow 0-1#$</td>
<td>$0$</td>
<td>$3\flat \leftrightarrow 3#$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Varieties</td>
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<td>diatomic/chronic</td>
<td>chronic</td>
<td>diatomic</td>
<td>chronic</td>
<td>chronic</td>
<td>chromatic</td>
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</tr>
<tr>
<td>Accretion</td>
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<td>yes</td>
<td>yes</td>
<td>no</td>
<td>yes</td>
<td>no</td>
<td>no</td>
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<td>yes</td>
</tr>
<tr>
<td>$\text{Inverse}$</td>
<td>$D^{-1}$</td>
<td>$F^{-1}$</td>
<td>$M^{-1}/m^{-1}$</td>
<td>$R/r$</td>
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<td>$P$</td>
<td>$A/\alpha$</td>
<td>$\Delta/\delta$</td>
<td>$\rho^{-1}$</td>
</tr>
<tr>
<td>Identity expr.</td>
<td>$D^{12}$</td>
<td>$F^{12}$</td>
<td>$M^3/m^3$</td>
<td>$R^2/r^2/R^{12}/r^{12}$</td>
<td>none</td>
<td>$P^2$</td>
<td>$A^2/\alpha^2$</td>
<td>$\Delta^2/\delta^2$</td>
<td>$\rho^3/\rho^3$</td>
</tr>
</tbody>
</table>